Energy
Market
Company

Stepping Through Co-Optimisation

A Market Clearing Engine Co-Optimisation Study

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About the Author

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Lu Feiyu joined Energy Market Company, the market operator for the National Electricity Market of Singapore, as a Market Analyst in March 2002. His primary responsibilities are the daily operation of the market and review of its outcome, dissemination of market information and in-house application development. Feiyu is also one of the company's pioneers in conducting local and international training and educational forums about the market clearing engine, specifically its formulations, pricing methodology and system enhancements. He is actively involved in enhancing the market system by identifying gaps between business processes and the market system, suggesting improvements and preparing and performing user acceptance tests. He also contributes to the market rule change process through technical reviews of the proposed changes.

Feiyu holds a Bachelor degree in Electrical Engineering from Tianjin University (China) and a Master of Engineering degree in Electrical and Electronic Engineering from Nanyang Technological University (Singapore).

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1.0 Introduction

The Market Clearing Engine (MCE) is a computer model that uses theoretically tight methods to determine optimal schedules that maximise the social welfare while meeting the system demand and satisfying the limits of generators, loads and the transmission system. The MCE employs an established technique called linear programming, which is a simultaneous solution method based on solving algebraic equations. It solves the constraints, limits and requirements as a set of simultaneous linear equations to find an energy, regulation and reserves schedule. Since there are many solutions to these equations (each being an implementable schedule), it chooses among them to find the one that maximises the objective function of the greatest social welfare. It simultaneously determines prices for all constraints reflecting the marginal cost of satisfying those constraints – called shadow prices (for example, the extra cost of another 1MW[1] of load at a node is the nodal energy price). In the National Electricity Market of Singapore (NEMS), the MCE solves the linear program model using market leading commercial software CPLEX.

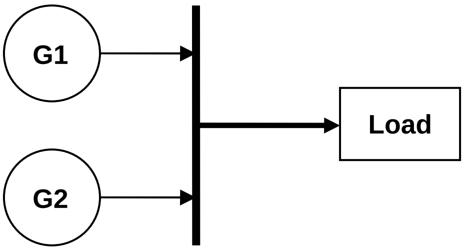
The MCE aims to find an optimal solution for energy, regulation and reserves as a whole. Hence, these products are always co-optimised in the MCE. Due to the complex nature of the NEMS model[2], the co-optimisation may not be straightforward and clear-cut. This paper endeavours to explore the co-optimisation aspect of the MCE, in a step-by-step approach. First, a simplified system is presented to show the effects of co-optimisation between energy and reserve. Though the system is imaginary, the basic logic employed reflects the essence of co-optimisation. Next, a more complicated case taken from the NEMS operation is presented, which focuses on manual price re-discovery of energy and regulation, thereby revealing the trade off between energy and regulation. Finally, a complete study is conducted on a real case with price spikes and heavy co-optimisation. In this study, the price is re-established using Right-Hand-Side (RHS) incremental analysis, which demonstrates the co-optimisation effect precisely
and accurately.

[1] Note that although *1MW* is used in the description, it should be interpreted as *1 incremental unit* which is infinitely small.
[2] NEMS is designed to simulate the physical system closely which leads to its complex nature. To name a few elements:

- It has not only energy and reserve markets, but a regulation market as well.
- Its reserve market takes into account the power system automatic response and has been fine-tuned to model the high load performance of the generation units.
- The reserve provisions are classified into groups, reflecting different levels of effectiveness.
- It models the transmission loss at the branch level and uses four segments to approximate the quadratic loss curve.

2.0 A Simplified Example (Energy–Reserve Trade Off)

As a simplified example, assume a single node or busbar system with two generation units (G1 and G2) and one load connected to this node. The load is kept at a constant value, while the system risk increases independent of the generation pattern. Ten offer blocks are submitted for each generating unit for each 30-minute trading period.



The generating unit characteristics and offer blocks for energy and reserve are as follows:

Energy	Block 1	Block 2	Block 3
G1 (Cap.=100MW)	60MW $20/MWh	40MW $40/MWh	-
G2 (Cap.=100MW)	100MW $25/MWh	-	-

Reserve	Block 1	Block 2
G1 (Max.=20MW)	20MW $2/MWh	-
G2 (Max.=100MW)	100MW $0/MWh	-

The system condition with increments of risk, which is the reserve requirement, is shown below with the resulting schedules and energy and reserve prices:

Load (MW)		140			
Risk (MW)		0	10	30	50
Energy Schedule (MW)	**G1**	60	60	60	70
	G2	80	80	80	70
Reserve Schedule (MW)	**G1**	0	0	10	20
	G2	0	10	20	30
Energy Price ($/MWh)		$25	$25	$27	$40
Reserve Price ($/MWh)		$0	$0	$2	$15

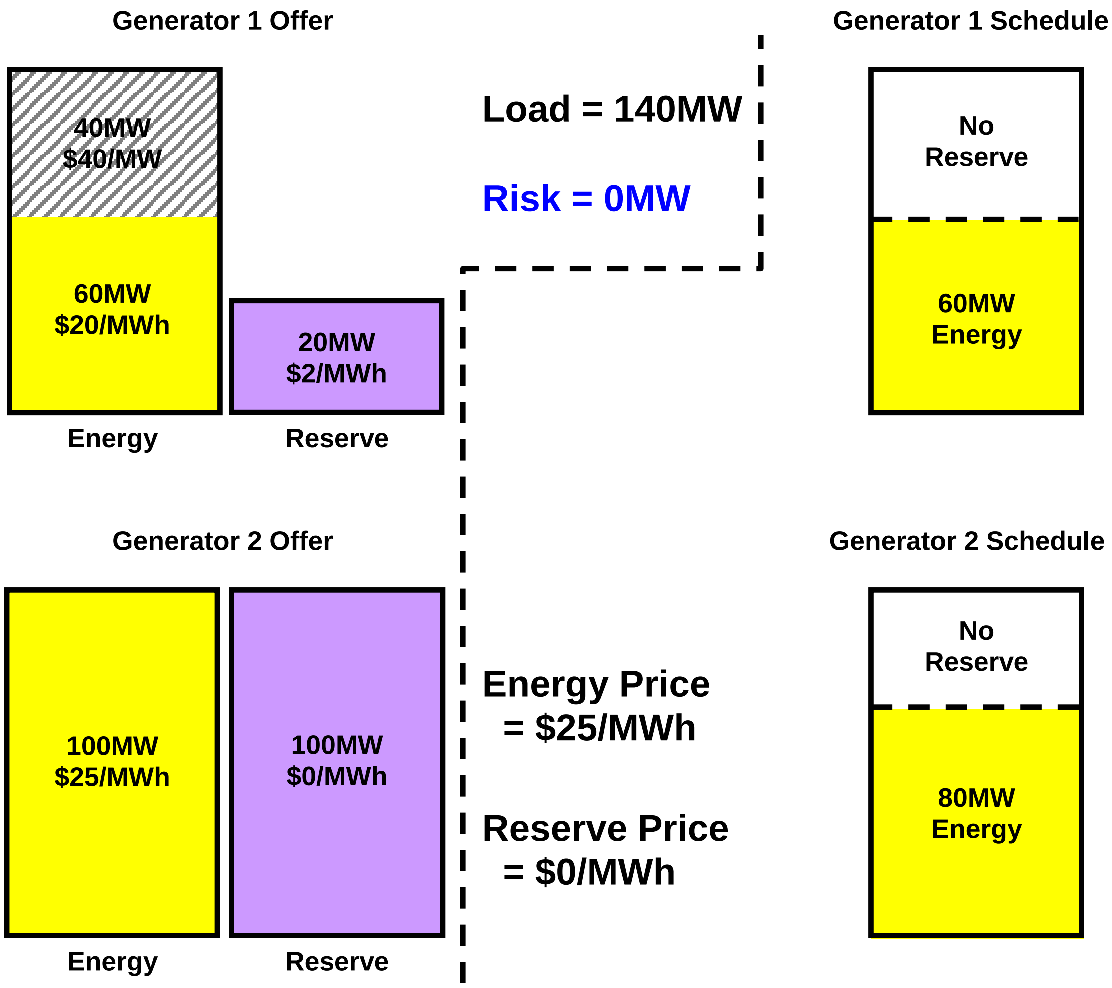
In the next four sections, each of the increments of risk or conditions is worked through.

2.1 Condition 1 (Risk = 0MW)

The load of 140MW is supplied by G1 (60MW) and G2 (80MW), based on merit order. Since the risk is 0MW, there is no dispatch of reserve from either unit.

The next MW of load increment would be supplied by the $25/MWh generation offer from unit G2. Hence the energy price is $25/MWh.

The next MW of risk increment would be covered by the $0/MWh reserve offer from G2. Hence the reserve price is $0/MWh.

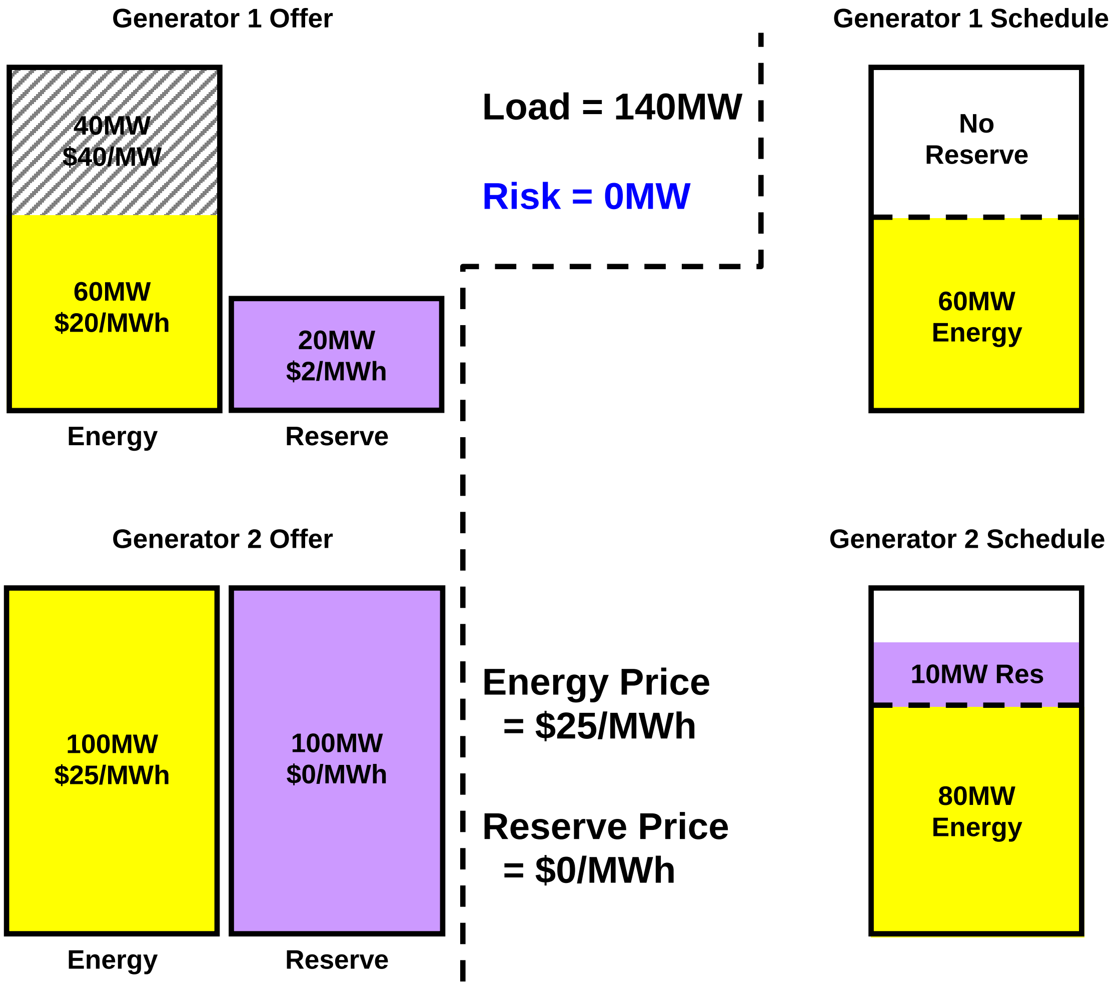


2.2 Condition 2 (Risk = 10MW)

The energy dispatch is the same as that for Condition 1, i.e., 60MW from G1 and 80MW from G2. Since the risk has been increased to 10MW, G2's reserve must be dispatched to pick up the demand, which is based on merit order.

The next MW of load increment would be supplied by the $25/MWh generation offer from unit G2. Hence the energy price is $25/MWh.

The next MW of risk increment would be covered by the $0/MWh reserve offer from G2. Hence the reserve price is $0/MWh.

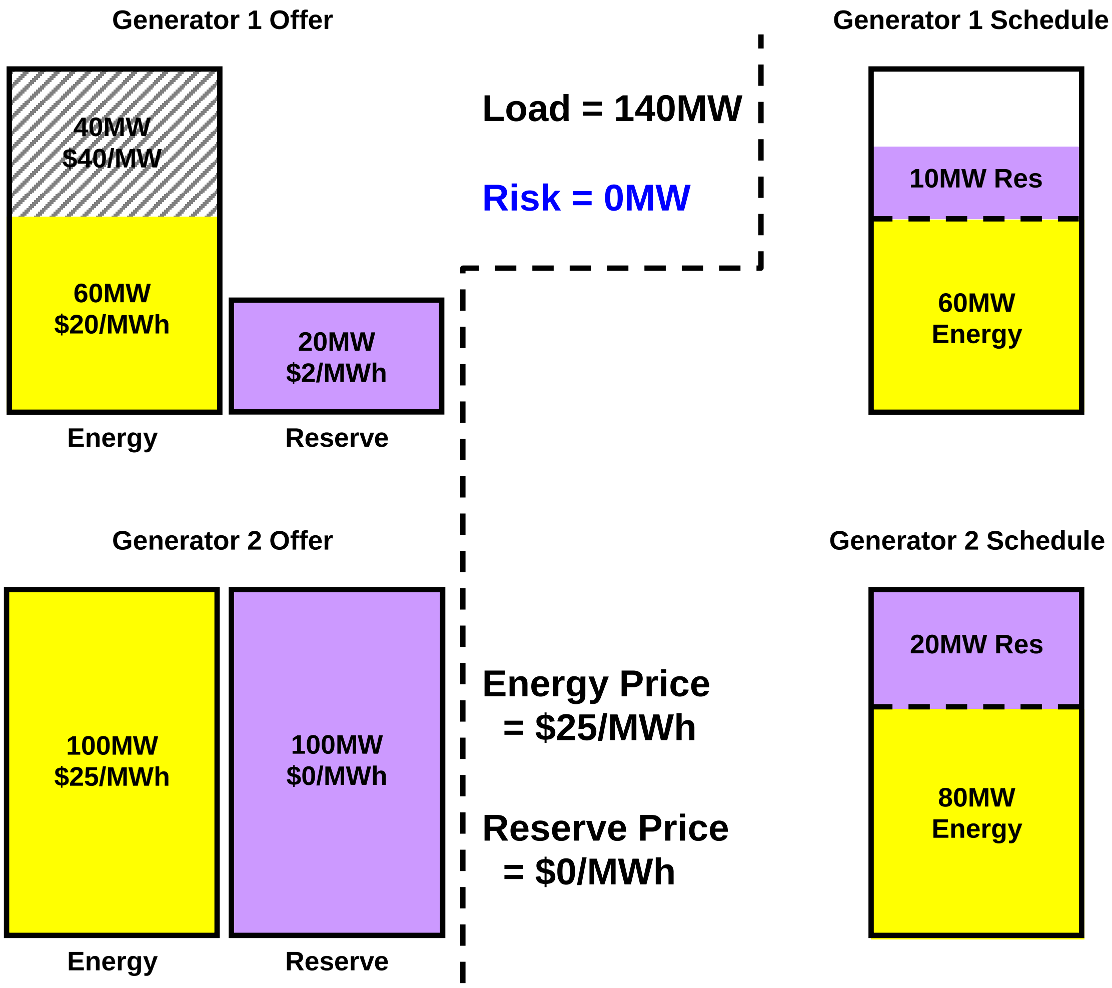


2.3 Condition 3 (Risk = 30MW)

The energy dispatch is the same as that for Conditions 1 and 2, i.e., 60MW from G1 and 80MW from G2. Since the risk has been increased further to 30MW, both G1 and G2 must dispatch reserve (10MW from G1 and 20MW from G2) to satisfy the demand. The reserve dispatch is no longer purely based on merit order[3], because G2's capacity has been fully utilised, which co-optimises energy and reserve. Hence, the energy price is not derived from the offers of Generator 1 or 2. It is the result of a trade off between energy and reserve.

The next MW of load increment would be covered by depressing G2's reserve schedule (saving $0/MWh) so as to avail the energy (at $25/MWh). However, G1 must dispatch more reserve (costing $2/MWh) to replace G2's decrement. The net cost is $27/MWh. Hence the energy price is $27/MWh.

The next MW of risk increment would be covered by the $2/MWh reserve offer from G1. Hence the reserve price is $2/MWh.



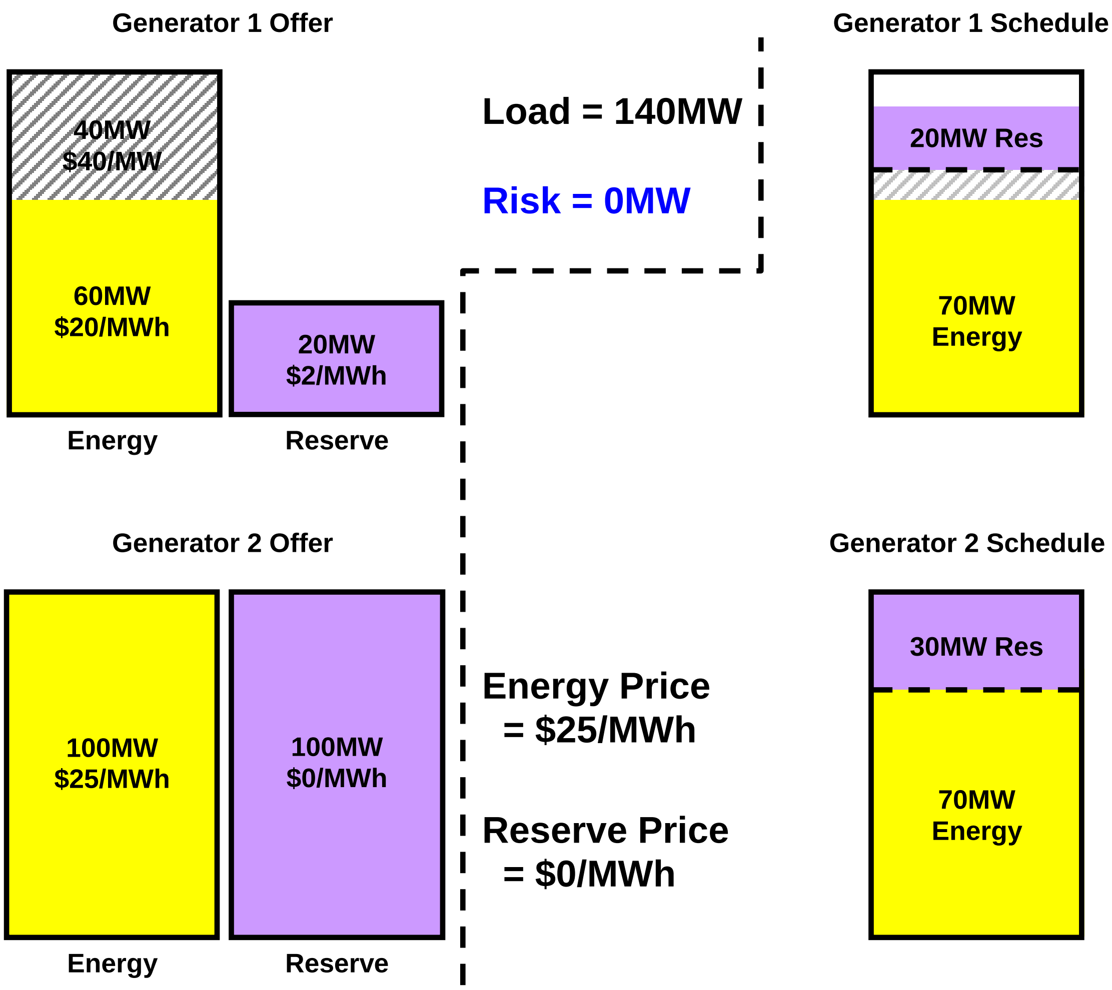
[3] Base on purely merit order, all 30MW of reserve should come from G2.

2.4 Condition 4 (Risk = 50MW)

With the risk further increased, the trade off between energy and reserve becomes more complex. This results in changes in energy dispatch; 70MW is dispatched from G1 and 70MW is from G2. All of the reserves offered by G1 are cleared in full and G2 is cleared of 30MW reserve. The reserve price is not derived from the offers of Generator 1 or 2. It is the result of trade offs between energy and reserve.

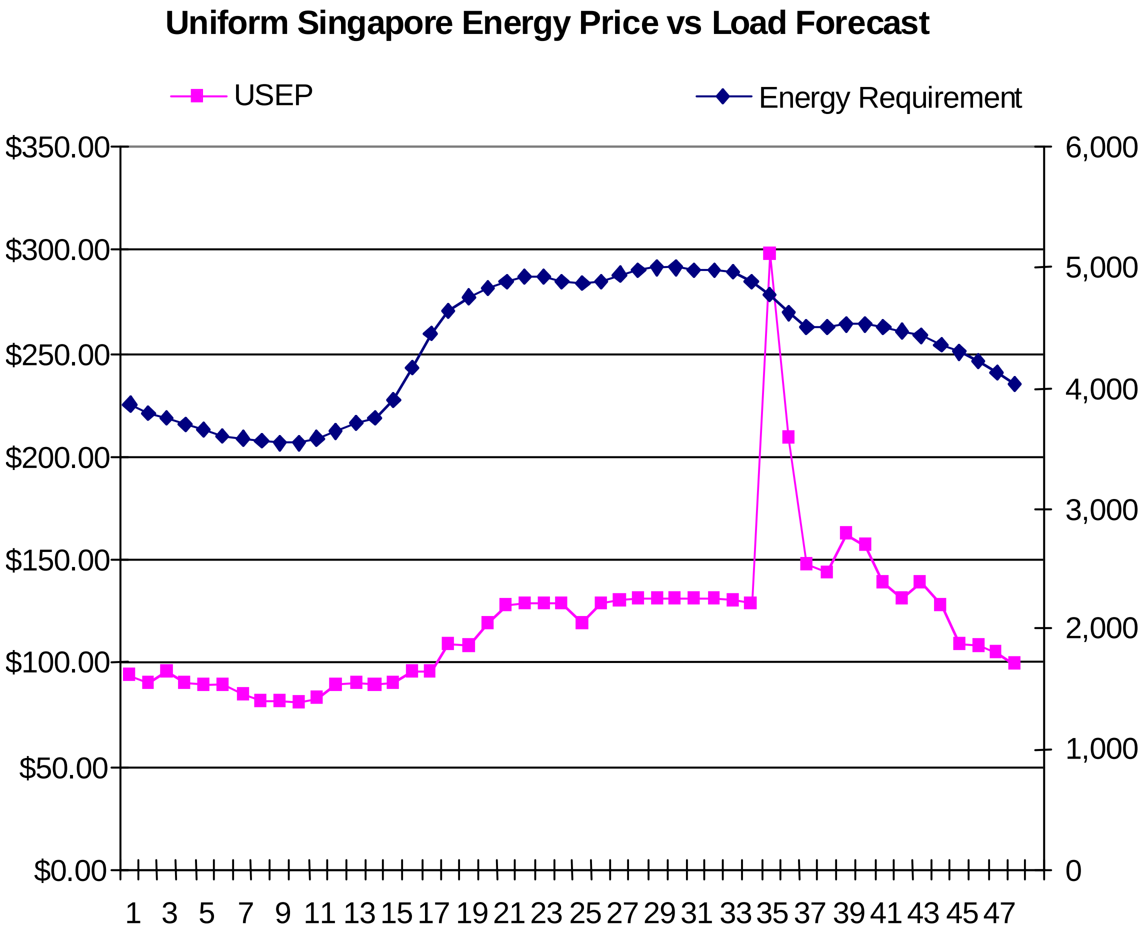
The next MW of load increment would be supplied by $40/MWh generation from unit G1. Hence the energy price is $40/MWh.

The next MW of risk increment would be covered by depressing G2's energy schedule (saving $25/MWh) so as to avail the reserve (at $0/MWh). However, G1 must dispatch more energy (costing $40/MWh) to replace G2's decrement. The net cost is $15/MWh. Hence the reserve price is $15/MWh.



3.0 A Real Case of Co-Optimisation (Energy and Regulation)

This case is taken from the NEMS. The Uniform Singapore Energy Price (USEP) is the weighted-average of the nodal prices at all off-take nodes. The USEP–Demand curve on one trading day is shown below:



A price spike was observed for period 35, during which the clearing prices were:

- USEP = $298.15/MWh
- Primary Reserve = $244.78/MWh
- Secondary Reserve = $15.79/MWh
- Contingency Reserve = $34.74/MWh
- Regulation = $294.58/MWh

The root cause of this price spike was found to be the tripping of a generating unit. However, the accuracy of the prices was still subject to justification. Following is a manual price re-discovery that was conducted for energy and regulation, which reveals the trade off between these two products.

3.1 Analysis of energy prices

The generating unit that provides the next MW of demand is called the marginal set. The nodal price for the energy marginal set cleared at $295.43/MWh. The generating unit was found to be offering its energy as below:

Energy Offer	Offer Quantity (MW)	Offer Price ($/MWh)	Cleared Quantity (MW)
Block 1	130	$0.99	130
Block 2	10	$90	10
Block 3	10	$93	10
Block 4	10	$95	10
Block 5	10	$99	10
Block 6	10	$109	10
Block 7	10	$118	10
Block 8	20	$137	20
Block 9	20	$250	1.449

This unit is bound by the Max. Capacity constraint on both secondary reserve and contingency reserve. Hence, its secondary reserve and contingency reserve offers are also extracted, as shown below:

Secondary Reserve Offer	Offer Quantity (MW)	Offer Price ($/MWh)	Cleared Quantity (MW)
Block 1	20	$0	20
Block 2	2	$1.16	0.551

Contingency Reserve Offer	Offer Quantity (MW)	Offer Price ($/MWh)	Cleared Quantity (MW)
Block 1	40	$0	20.551

The price at any node is determined by the cost of another 1MW of load at this node. In this case, if the nodal demand increases by 1MW, the energy cost of this 1MW is $250 as per its offer.

Since this generating unit is bound by the Max. Capacity constraint for both secondary reserve and contingency reserve, as more energy is dispatched it reduces the availability of secondary and contingency reserves. Hence, the 1MW energy increment will depress its secondary and contingency reserve dispatch by 0.85MW and 0.95MW, respectively[4]. The offer price from this unit for secondary reserve is $1.16, and $0 for contingency reserve. Therefore, the cost for the replacement of secondary reserve is:

($15.79 - $1.16) * 0.85 = $12.44

Similarly, the cost for the replacement of contingency reserve is:

($34.74 - $0) * 0.95 = $33

Summing the above costs, we obtain the total cost of this 1MW nodal demand increase. That is:

$250 + $12.44 + $33 = $295.44

This is close[5] to the system calculated MNN price ($295.43/MWh), which proves that the MCE works as expected.

[4] The reduction in reserves is not 1MW due to the reserve effectiveness group of this generating unit. Because the secondary reserve of this unit belongs to group B and its contingency reserve belongs to group A, its effective secondary and contingency reserves are only 0.85 and 0.95, respectively.

[5] A more accurate and precise price re-discovery (via RHS) is introduced in section 4.

3.2 Analysis of regulation prices

The cleared regulation price is $294.58/MWh, but the regulation marginal set (which is different from the energy marginal set) is offered at only $150/MWh, as shown below:

Regulation Offer	Offer Quantity (MW)	Offer Price ($/MWh)	Cleared Quantity (MW)
Block 1	1	$0.01	1
Block 2	2	$27.00	2
Block 3	2	$100.00	2
Block 4	2	$150.00	1

This unit is bound by the Max. Capacity constraint. Hence, its energy offer is also extracted, as shown below:

Energy Offer	Offer Quantity (MW)	Offer Price ($/MWh)	Cleared Quantity (MW)
Block 1	135	- $14.0	135
Block 2	40	$75.4	40
Block 3	1	$87.0	1
Block 4	1	$91.0	1
Block 5	1	$95.5	1
Block 6	5	$108.0	5
Block 7	37	$130.0	37
Block 8	14	$150.0	6

(The cleared MNN price for this unit is $294.58/MWh)

The regulation price is also determined by the cost of another 1MW of demand. In this case, if the regulation demand increases by 1MW, the cost of providing this extra MW of regulation is $150 as per the offer.

Since this unit is bound by the Max. Capacity constraint, as more regulation is dispatched it reduces the availability of energy. Hence, the 1MW regulation increment will depress its energy dispatch by 1MW[6]. The MNN price for this unit is $294.58/MWh. Therefore, the cost for the replacement of its energy is:

$294.58 - $150 = $144.59

Summing up the above costs, we can obtain the total cost of this 1MW regulation demand increment. That is:

$150 + $144.59 = $294.58

This is exactly the same as the system calculated regulation price ($294.58/MWh), which proves that the MCE works as expected.

[6] Unlike the reserves, regulation has a one-to-one replacement with energy.

3.3 Analysis of reserve prices

The cleared prices for the three classes of reserve can theoretically be derived using the same method as shown in the previous sections. However, due to the complexity of the reserve model and the interaction between the various reserve classes, the process of manual price re-discovery is tedious, if not impossible. In section 4, we demonstrate the price discovery of all three classes of reserve.

4.0 A Complete Case Study (Energy, Regulation and Reserve)

A real case of price spikes on energy, regulation and two classes of reserve is the subject of this study. Due to the heavy co-optimisation observed in this case, a manual price re-discovery is almost impossible. Hence, a well established right-hand-side (RHS) incremental analysis is employed, which requires a re-run of the case with the RHS increment on the equation of interest. Note that damping generating units require higher reserve.

The system result reported by the original real-time dispatch run is as follows:

Product	Requirement (MW)	Cleared (MW)	Price ($/MWh)
Energy	4664.325	4690.833	$204.00
Regulation	100	100	$71.62
Primary Reserve	345.927	345.927	$121.56
Secondary Reserve	352.014	352.014	$53.58
Contingency Reserve	380	380	$0.96

After preliminary investigation, the market mechanism and information for this period are summarised as below:

Product	Marginal Set*	Offer Price ($/MWh)	Marginal Price ($/MWh)	Damping Gen?
Energy	G11	$200	$201.82	Y
Regulation	G8	$0	$71.62	N
Primary Reserve	G5	$67.07	$121.56	N
Secondary Reserve	G14	$20	$53.58	N
Contingency Reserve	G10	$0.96	$0.96	Y

*Gx denotes a specific generating unit

Further investigation on each product price is conducted using RHS analysis.

4.1 Analysis of energy price

A re-run is performed with the RHS of the Node Balance Generation Constraint[7] for G11 incremented from 0MW to 0.01MW (effectively an incremental increase in nodal demand). The dispatch schedule changes are listed below:

Unit & Product	Change in MW	Offer Price ($/MWh)	Change in Cost for .01MW
G5 ~ Energy ~ PriRes (MS) ~ SecRes ~ ConRes	-0.000199 0.000199 (Effectiveness = 0.75) 0.000199 (Effectiveness = 0.75) 0.000199 (Effectiveness = 0.95)	$130.04 $50.30 $10.30 $0.00	-$0.0259 $0.0100 $0.0021 $0.0000
G10 ~ ConRes (MS)	-0.000199 (Effectiveness = 0.95)	$0.91	-$0.0002
G11 ~ Energy (MS)	0.009969	$200.00	$1.9938
G14 ~ Energy ~ SecRes (MS)	0.000230 -0.000230 (Effectiveness = 0.65)	$180.00 $13.00	$0.0414 -$0.0030
			Sum = $2.0182

Marginal Set (MS)

Though the dispatch results are all derived in one solver run, the process can be broken down into the following sequence for easier understanding:

1. Nodal demand at G11 goes up by 0.01MW, as found through RHS analysis.
2. G11 is the marginal set for energy and hence its energy dispatch is increased by 0.01MW to meet the increment in the nodal demand. However, G11 is a damping generator with a factor of 0.015 and hence its energy increment drives the risk for primary reserve up by 0.00015MW (= 0.01MW * 0.015).
3. G5 is the marginal set for primary reserve and hence its primary reserve dispatch is increased by 0.0002MW (= 0.00015MW / 0.75) to meet the increment in system primary risk. However, due to the Max. Capacity constraint, the energy from G5 is depressed by 0.0002MW, while the secondary and contingency reserve take the opportunity to increase by 0.0002MW.
4. a) G14 is the marginal set for secondary reserve and hence its secondary reserve dispatch is decreased by 0.0002307MW (= 0.0002 * 0.75 / 0.65)
   b) G10 is the marginal set for contingency reserve and hence its contingency reserve dispatch is decreased by 0.0002MW (= 0.0002 * 0.95 / 0.95)
5. G14 is also constrained by Max. Capacity and hence its secondary reserve reduction makes room for its energy dispatch up by 0.0002307MW.
6. The energy dispatch increment on G14 relieves some energy burden on the marginal set (G11) so that the energy dispatch from G11 can be reduced.
7. The above process iterates until a balance is achieved, i.e., the net energy increment in the market meets the RHS increment at G11, as shown in the table.
8. All of the changes in cost are summed to be $2.0182, which is caused by the 0.01MW increment of the nodal demand. Hence, the nodal energy price for G11 is $201.82/MWh.

7 $$\sum_{g \in \mathrm{OFFERS}_n} \mathrm{Generation}_g - \sum_{p \in \mathrm{BIDS}_n} \mathrm{Purchase}_p + \sum_{j \in \mathrm{DEFICITGENERATIONBLOCKS}_n} \mathrm{DeficitGenerationBlock}_{n,j} - \sum_{j \in \mathrm{EXCESSGENERATIONBLOCKS}_n} \mathrm{ExcessGenerationBlock}_{n,j} - \mathrm{NodeNetInjection}_n = 0$$

4.2 Analysis of regulation price

A re-run is performed with the RHS of the Regulation Balance Constraint[8] for G11 incremented from 0MW to 0.1MW (effectively an incremental increase in nodal demand). The dispatch schedule changes are listed in the table below:

Unit & Product	Change in MW	Offer Price ($/MWh)	Change in Cost for 0.1MW
G8 ~ Energy ~ Regulation (*MS*)	-0.100000 0.100000	$130.08 $0.00	-$13.0080 $0.0000
G5 ~ Energy ~ PriRes (*MS*) ~ SecRes ~ ConRes	-0.001993 0.001993 (Effectiveness = 0.75) 0.001993 (Effectiveness = 0.75) 0.001993 (Effectiveness = 0.95)	$130.04 $50.30 $10.30 $0.00	-$0.2591 $0.1002 $0.0205 $0.0000
G10 ~ ConRes (*MS*)	-0.001993 (Effectiveness = 0.95)	$0.91	-$0.0018
G11 ~ Energy (*MS*)	0.099630	$200.00	$19.9260
G14 ~ Energy ~ SecRes (*MS*)	0.002299 -0.002299 (Effectiveness = 0.65)	$180.00 $13.00	$0.4138 -$0.0299
			Sum = $7.161824

MS: Marginal Set

Though the dispatch results are all derived in one solver run, the process can be broken down into the following sequence for easier understanding:

1. Regulation demand goes up by 0.1MW, as specified by RHS analysis.
2. G8 is the marginal set for regulation and hence its regulation dispatch is increased by 0.1MW to pick up the requirement increment. However, due to the Max. Capacity constraint, the energy from G8 is depressed down by 0.1MW.
3. G11 is the marginal set for energy and hence its energy dispatch is increased by 0.1MW to cover the lost energy from G8.
4. G11 is a damping generator with a damping factor of 0.015 and hence its energy increment drives the risk for primary reserve up by 0.0015MW (= 0.1MW * 0.015).
5. G5 is the marginal set for primary reserve and hence its primary reserve dispatch is increased by 0.002MW (= 0.0015MW / 0.75) to meet the increment in risk for primary reserve. However, due to the Max. Capacity constraint, the energy from G5 is depressed by 0.002MW, while the secondary and contingency reserves take the opportunity to increase by 0.002MW.
6. a) G14 is the marginal set for secondary reserve and hence its secondary reserve dispatch is decreased by 0.002308MW (= 0.002 * 0.75 / 0.65).
   b) G10 is the marginal set for contingency reserve and hence its contingency reserve dispatch is decreased by 0.002MW (= 0.002 * 0.95 / 0.95).
7. G14 is also constrained by Max. Capacity, and hence its secondary reserve reduction makes room for its energy dispatch up by 0.002308MW.
8. The energy dispatch increment on G14 relieves some energy burden on the marginal set, i.e., G11. Hence, the energy dispatch from G11 can be reduced.

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[8] $$\sum_{l \in \mathrm{REGULATIONOFFERS}} \text{Regulation}_l + \text{DeficitRegulation} - \text{RegulationRequirement} \geq 0$$

[10] $$\sum_{x \in \mathrm{RESERVEGROUPS}_c} \text{EffectiveReserve}_x + \text{DeficitReserve}_c - \text{Risk}_c \geq 0$$

9. The above process iterates until a balance is achieved, i.e., the net changes in all markets are zero except a 0.1MW increment in regulation market, as shown in the table.
10. All of the changes in cost are summed to be $7.161824, which is caused by the 0.1MW increment of the regulation demand. Hence, the regulation price is $71.62/MWh.

4.3 Analysis of primary reserve price

A re-run is performed with the RHS of the Primary Reserve Balance Constraint[10] incremented from 0MW to 0.1MW (effectively an incremental increase in nodal demand). The dispatch schedule changes are listed in the table below:

Unit & Product	Change in MW	Offer Price ($/MWh)	Change in Cost for 01.MW
G5 ~ Energy ~ PriRes (*MS*) ~ SecRes ~ ConRes	-0.132923 0.132923 (Effectiveness = 0.75) 0.132923 (Effectiveness = 0.75) 0.132923 (Effectiveness = 0.95)	$130.04 $50.30 $10.30 $0.00	-$17.2853 $6.6860 $1.3691 $0.0000
G10 ~ ConRes (*MS*)	-0.132923 (Effectiveness = 0.95)	$0.91	-$0.1210
G11 ~ Energy (*MS*)	-0.020531	$200.00	-$4.1062
G14 ~ Energy ~ SecRes (*MS*)	0.153373 -0.153373 (Effectiveness = 0.65)	$180.00 $13.00	$27.6071 -$1.9938
			Sum = $12.156

MS: Marginal Set

Though the dispatch results are all derived in one solver run, the process can be broken down into the following sequence for easier understanding:

1. Primary risk goes up by 0.1MW, as specified by RHS analysis.
2. G5 is the marginal set for primary risk. Hence, it picks up the increment by providing 0.133333 (= 0.1MW / 0.75) more MW of primary reserve. However, due to the Max. Capacity constraint, the energy from G5 is depressed by 0.133333MW, while the secondary and contingency reserves take the opportunity to increase by 0.133333MW.
3. a) G14 is the marginal set for secondary reserve and hence its secondary reserve dispatch is decreased by 0.153846MW (=0.133333 * 0.75 / 0.65).
   b) G10 is the marginal set for contingency reserve and hence its contingency reserve dispatch is decreased by 0.133333MW (=0.133333 * 0.95 / 0.95).
4. G14 is also constrained by Max. Capacity, and hence its secondary reserve reduction makes room for its energy dispatch up by 0.153846MW.
5. More energy (0.153846MW) is dispatched from G14, while less energy (0.133333MW) is dispatched from G5. The net effect is more energy (0.020513MW[11]) available in the market. Since G11 is the marginal set in energy market, its dispatch is depressed by 0.020513MW.
6. G11 is a damping generator with a damping factor of 0.015 and hence its energy reduction drives the risk for primary reserve down by 0.000308MW (=0.020513MW * 0.015). This relieves primary reserve requirement. Hence, the primary reserve dispatch from G5 can be reduced.
7. The above process iterates until a balance is achieved, i.e., the net changes in all markets are zero except a 0.1MW increment in the primary reserve market, as shown in the table.
8. All of the changes in cost are summed to be $12.156, which is caused by the 0.1MW increment of the primary risk. Hence, the primary reserve price is $121.56/MWh

[11] Assuming no transmission losses, for the sake of simplicity.

[13] $$\sum_{x \in \text{RESERVEGROUPS}_c} \text{EffectiveReserve}_x + \text{DeficitReserve}_c - \text{Risk}_c \geq 0$$

4.4 Analysis of secondary reserve price

A re-run is performed with the RHS of the Secondary Reserve Balance Constraint[13] incremented from 0MW to 0.05MW (effectively an incremental increase in nodal demand). The dispatch schedule changes are listed in the table below:

Unit & Product	Change in MW	Offer Price ($/MWh)	Change in Cost for 0.05MW
G5 ~ Energy ~ PriRes (*MS*) ~ SecRes ~ ConRes	-0.001534 0.001534 (Effectiveness = 0.75) 0.001534 (Effectiveness = 0.75) 0.001534 (Effectiveness = 0.95)	$130.04 $50.30 $10.30 $0.00	-$0.1994 $0.0771 $0.0158 $0.0000
G10 ~ ConRes (*MS*)	-0.001534 (Effectiveness = 0.95)	$0.91	-$0.0014
G11 ~ Energy (*MS*)	0.076686	$200.00	-$15.3373
G14 ~ Energy ~ SecRes (*MS*)	-0.075153 0.075153 (Effectiveness = 0.65)	$180.00 $13.00	-$13.5276 $0.9770
			Sum = $2.6788

MS: Marginal Set

Though the dispatch results are all derived in one solver run, the process can be broken down into the following sequence for easier understanding:

1. Secondary risk goes up by 0.05MW, as specified by RHS analysis.
2. G14 is the marginal set for secondary risk. Hence, it picks up the increment by providing 0.076923 (= 0.05MW / 0.65) more MW of secondary reserve. However, due to the Max. Capacity constraint, the energy from G14 is depressed by 0.076923MW.
3. Since G11 is the marginal set in energy market, it has to be dispatched more (0.076923MW[14]) to cover the lost energy from G14.
4. G11 is a damping generator with a damping factor of 0.015 and hence its energy increment drives the risk for primary reserve up by 0.001154MW (=0.076923MW * 0.015).
5. G5 is the marginal set for primary reserve and hence its dispatch is increased by 0.001538MW (=0.001154MW / 0.75) to meet the increment in system primary risk. However, due to the Max. Capacity constraint, the energy from G5 is depressed by 0.001538MW, while the secondary and contingency reserves take the opportunity to increase by 0.001538MW.
6. a) G10 is the marginal set for contingency reserve and hence its contingency reserve dispatch is decreased by 0.001538MW (=0.001538 * 0.95 / 0.95).
   b) The secondary reserve dispatch increment from G5 relieves the burden on G14. Hence, the secondary reserve dispatch from G14 can be reduced.
7. The above process iterates until a balance is achieved, i.e., the net changes in all markets are zero except a 0.05MW increment in secondary reserve market, as shown in the table.
8. All of the changes in cost are summed to be $2.6788, which is caused by the 0.05MW increment of the secondary risk. Hence, the secondary reserve price is $ 53.58/MWh

[14] Assuming no transmission losses for simplicity.

[16] $$\sum_{x \in \text{RESERVEGROUPS}_c} \text{EffectiveReserve}_x + \text{DeficitReserve}_c - \text{Risk}_c \geq 0$$

4.5 Analysis of contingency reserve price

A re-run is performed with the RHS of the Contingency Reserve Balance Constraint[16] incremented from 0MW to 0.1MW (effectively an incremental increase in nodal demand). The dispatch schedule changes are listed in the table below:

Unit & Product	Change in MW	Offer Price ($/MWh)	Change in Cost For 0.1MW
G10 ~ ConRes *(MS)*	0.105263	$0.91	$0.0958

MS: Marginal Set

In this case, the cleared price is directly from the offer. Due to the contingency reserve effectiveness of G10 being 0.95, the contingency reserve price is $0.91 / 0.95 = $0.96/MWh

4.6 Conclusion

From the complete analysis of this complex case, it can be seen that the co-optimisation process can be very complicated and it is not always possible to reach a conclusion from the apparent evidence gathered. The manual price re-discovery process may work in simple cases, by employing the basic idea of co-optimisation. However, an accurate and concrete answer can only be found with a re-run using the RHS increment. Due to the time constraint in employing the RHS approach, it is not recommended to conduct such an analysis on each real-time dispatch run.

Nevertheless, the results from this price re-discovery exercise provide strong justification that the prices calculated by the Market Clearing Engine are consistent with the formulation and content of the Market Rules. The co-optimisation works as expected.