Energy
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Nodal Price Difference by Transmission Loss

A Technical Study of the NEMS

By

Lu Feiyu

Senior Market Analyst

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About the Author

Lu Feiyu, Senior Market Analyst

Lu Feiyu joined Energy Market Company, the market operator for the National Electricity Market of Singapore, as a Market Analyst in March 2002. His primary responsibilities are the daily operation of the market and review of its outcome, dissemination of market information and in-house application development. Feiyu is also one of the company's pioneers in conducting local and international training and educational forums about the market clearing engine, specifically its formulations, pricing methodology and system enhancements. He is actively involved in enhancing the market system by identifying gaps between business processes and the market system, suggesting improvements and preparing and performing user acceptance tests. He also contributes to the market rule change process through technical reviews of the proposed changes.

Feiyu holds a Master of Engineering degree in Electrical and Electronic Engineering from Nanyang Technological University (Singapore).

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1.0 Introduction

In the past decade, a new wave of deregulation and restructuring has affected the electricity industry throughout the world. Among all of the changes, one of the most revolutionary is the pricing mechanism, which has evolved from fixed price to time-of-use pricing to today's mechanism of place-of-use pricing. The place-of-use price, or locational marginal price (LMP) as it is referred to, has become part of the standard market design adopted by the Federal Energy Regulatory Commission (FERC) in the United States.

The greatest benefit brought by LMP is that it uses market forces to manage transmission loss and congestion, by creating price signals reflecting the time and locational value of electricity.

LMP reflects the cost of generation and transmission to a particular location. Each busbar within the transmission grid has its own energy price for a dispatch interval. The price is determined by calculating the cost of serving an increment of load at that location, given the prices offered by generators (and the prices bid by dispatchable loads, if any) and given the transmission constraints that limit flows on the grid. The economic signals generated from LMP provide a useful guide for new investment in the future.

Each LMP reflects the incremental cost of re-dispatching the system to supply one more MW of load at a given location. Thus, LMP also provides a precise market-based method for pricing any re-dispatch required to relieve congestion.

While some jurisdictions use LMP in a an aggregated way, such as regional or zonal price, the NEMS models the transmission system at the circuit level and is therefore able to employ a nodal price regime. Each node (or bus) in the Singapore transmission grid is identified as a unique location. The price of energy is calculated at each of the connected nodes for any dispatch interval.

What makes the nodal price stand out is not the price itself, but the price differences among nodes. Two factors contribute to such differences. Not surprisingly, they are:

- transmission loss and
- transmission congestion.

Transmission congestion may cause price separation and/or a spring washer effect[1]. However, the most fundamental reason for the majority of price differences is transmission loss. Hence, this paper focuses on the relationship between nodal prices and transmission loss. Due to the technical nature of the topic, some electrical and mathematical background will help the reader's comprehension.

[1] More details can be found in another paper by the author titled *Spring Washer Effect*.

2.0 Loss Model

In the simplest way, the loss and flow relation can be described by a mathematical model.

[Notation: L - loss, F - flow, R - resistance, V - voltage, I - current]

Transmission loss is caused by the resistance in the transmission line or transformer, which is normally represented by the formula:

$$L = I^2 * R \quad (2.1)$$

Since the current can be derived via flow and voltage in a relationship of *I = F / V*, the equation (2.1) can be rewritten to:

$$L = (F^2 / V^2) * R$$

$$= F^2 * R / V^2 \quad (2.2)$$

Power system equations are often expressed in a 'per unit' format, for various reasons. The market clearing engine (MCE) of the NEMS is no exception; it uses the per unit system for the resistance and reactance of branch[2] data. Hence, the equation (2.2) can be rewritten to:

[Notation: pu - per unit value, b - base value]

$$L = F^2 * (R_{(pu)} * R_{(b)}) / (V_{(pu)} * V_{(b)})^2$$

$$= F^2 * R_{(pu)} * R_{(b)} / (V_{(pu)}{}^2 * V_{(b)}{}^2) \quad (2.3)$$

Since $R_{(b)} = V_{(b)}{}^2 / F_{(b)}$, the above formula becomes

$$L = F^2 * R_{(pu)} * (V_{(b)}{}^2 / F_{(b)}) / (V_{(pu)}{}^2 * V_{(b)}{}^2)$$

$$= F^2 * R_{(pu)} / (F_{(b)} * V_{(pu)}{}^2) \quad (2.4)$$

Since $V_{(pu)}$ = 1, the equation (2.4) can be rewritten to:

$$L = F^2 * R_{(pu)} / F_{(b)} \quad (2.5)$$

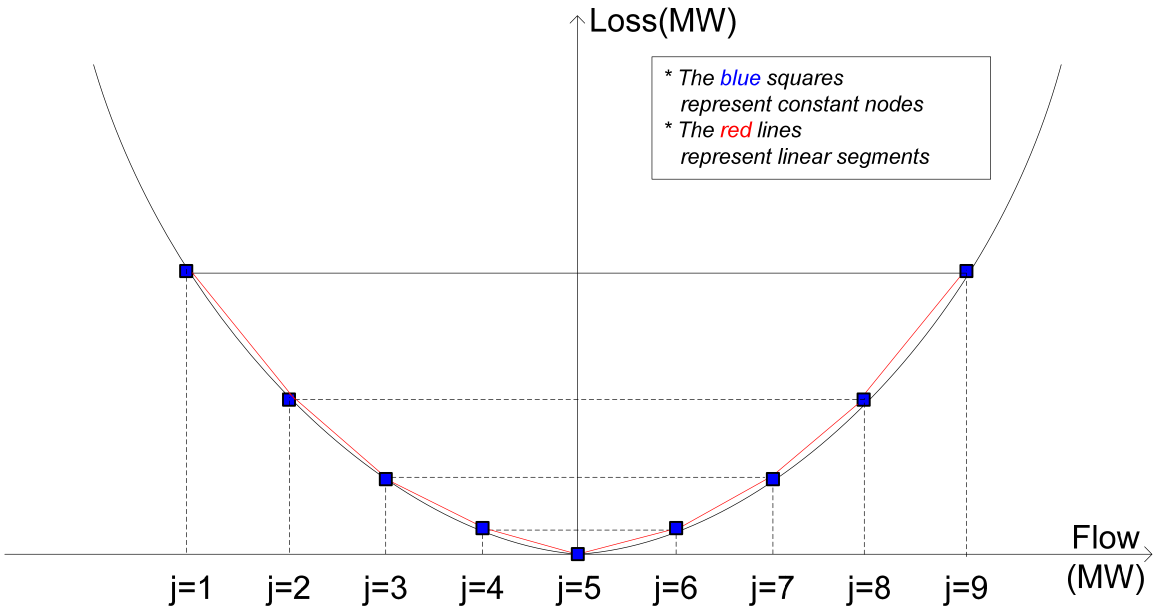
The NEMS uses 100 as the MVA base, and therefore the above formula can be simplified to:

$$L = F^2 * R_{(pu)} / 100 \quad (2.6)$$

The resistance values of all of the transmission lines and transformers in Singapore's power grid have been submitted as part of the standing data registration (in the per unit format). Hence, the flow and loss relationship can be readily derived using the above formula. However, this relationship is a quadratic curve that cannot be solved using linear programming. Hence, the

[2] Branch refers to either a transmission line or a transformer.

MCE approximates the quadratic curve using nine constant nodes and eight linear segments, as depicted in the diagram below:



The formula (2.6) is used by the MCE to calculate the nine constant nodes, and then the loss curve is drawn by connecting these nine nodes together. This process forms the eight linear segments, each of which can be represented by the following linear model:

$$L = k * F + c \qquad (2.7)$$

where k *is the slope of the linear segment, while* c *is the intersection with the* Y *axis.*

Once the flow-loss curve is available, it's not difficult to find the nodal price relationship across any transmission branch. However, one common misunderstanding is to use the segment slope, i.e., *k*, directly in the derivation of the price difference across the branch, as below:

$$P_{receiving} = P_{sending} * (1 + Slope_{segment(i)})$$

Theoretically correct, this approach does not replicate the exact answer that the MCE tells us. Hence, we need to explore the modelling details of the transmission system in the MCE so as to reveal the right formula to use.

3.0 Nodal Price Derivation

The MCE model employs a piece-wise linear approximation of the loss function with losses shared equally at the sending and receiving ends of the branch and flows measured at the mid-point of the branch. This is a well accepted industry practice.

As mentioned in the introduction, the energy price at each node can be derived by calculating the cost of serving an increment of load at that location. Using mathematical modelling language, this relationship can be expressed in the following way:

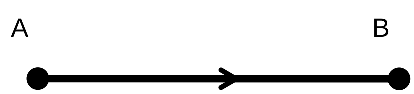
$$P_n = P_m * \left(1 + \frac{\partial TotalLoss}{\partial Demand_n}\right) \qquad (3.1)$$

where ‘n’ denotes any node n, while ‘m’ denotes marginal node

In the specific case of determining the price differential between two nodes (node A and node B) across a spur line, this general equation reduces to the following:

$$P_B = P_A * \left(1 + \frac{\partial l_{A-B}}{\partial f_B}\right) \qquad (3.2)$$

where ‘A’ is the sending node, ‘B’ is the receiving node, f_B is the flow into B from line A–B, and l_{A-B} is the loss on line A–B



Introducing the flow at the mid-point of line A–B as f_{A-B}, we can represent the derivative $\frac{\partial l_{A-B}}{\partial f_B}$ as the product of two more convenient derivatives as follows:

$$\frac{\partial l_{A-B}}{\partial f_B} = \frac{\partial l_{A-B}}{\partial f_{A-B}} * \frac{\partial f_{A-B}}{\partial f_B} \qquad (3.3)$$

Since the loss function employed is a linear function of the flow at the mid-point of the branch, we can borrow the formula (2.7) from Section 2:

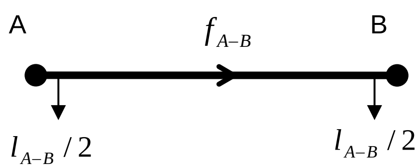
$$L = k * F + c \qquad (2.7)$$

Taking the first derivative of (2.7) we get:

$$\frac{\partial l_{A-B}}{\partial f_{A-B}} = k \qquad (3.5)$$

As defined by the model, the flow from line A–B into B is as follows:

$$f_B = f_{A-B} - l_{A-B}/2 \qquad (3.4)$$



Taking the first derivative of (3.4) we get:

$$\frac{\partial f_B}{\partial f_{A-B}} = 1 - k/2 \qquad (3.6)$$

Taking the reciprocal of (3.6) we get:

$$\frac{\partial f_{A-B}}{\partial f_B} = 1 / (1 - k/2) \qquad (3.7)$$

Substituting (3.5) and (3.7) into (3.3) and rearranging the formula, we get:

$$\frac{\partial l_{A-B}}{\partial f_B} = \frac{\partial l_{A-B}}{\partial f_{A-B}} * \frac{\partial f_{A-B}}{\partial f_B} = k / (1 - k/2) = 2k / (2 - k) \qquad (3.8)$$

Substituting (3.8) into (3.2) and rearranging the formula again, we get:

$$P_B = P_A * (1 + \frac{\partial l_{A-B}}{\partial f_B})$$

$$= P_A * (1 + 2k / (2 - k))$$

$$= P_A * (2 - k + 2k) / (2 - k)$$

$$= P_A * (2 + k) / (2 - k)$$

This is the equation that should be applied to the nodal price calculations across a radio circuit.

In a broader view, this formulation plays a critical role in the clearance mechanism of the energy market. It substantiates the realisation of the merit order dispatch (MOD). As the branches, i.e., transmission lines and transformers, transmit the generation from scattered generators to a centralised and interconnected grid, this formulation translates the generator offers to the central platform as well. Thus the offer prices can be compared and the merit order derived.

For the market participants, the understanding of this formulation helps their ad hoc studies. Gencos can apply this formulation to their unit transformers and thus understand the price difference between their market network nodes (MNN) and the grid. In addition, should they wonder why their units have different MNN prices though they are physically at the same station, this formulation enables them to find out the answer.

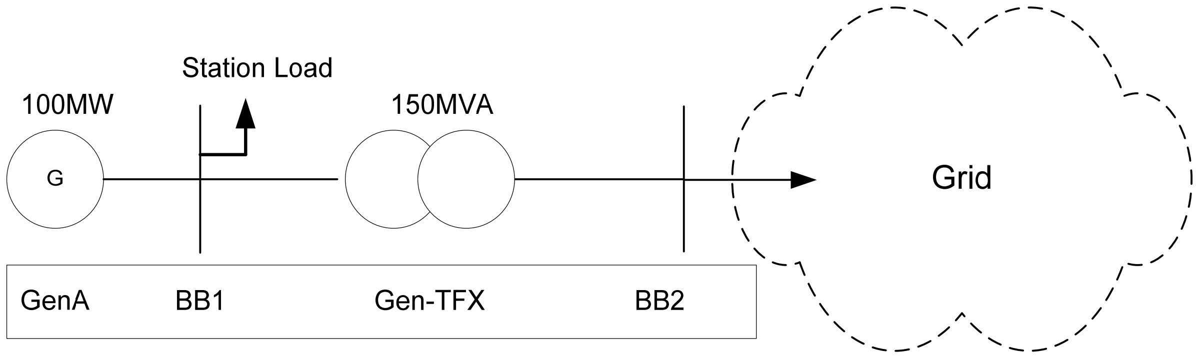
This formulation also implies another important characteristic about the nodal pricing regime: nodal prices increase along the power flow. In other words, the nodal price at the receiving end is always higher than that at the sending end. This explains why the gencos at different locations receive different MNN prices. Generally speaking,

- The nearer the generating unit is to the load centre, the higher the nodal price that it will get.
- The further away the generating unit is from the load centre, the lower the nodal price that it will get.

Though it seems that gencos near the load centre are rendered a natural advantage due to their locational superiority, the operating costs (such as rental and fuel supply) at such locations is generally higher, which justifies such nodal price advantages.

4.0 An Example

A certain genco has a generating unit, GenA, connected to the grid via transformer TFX, as below:



The standing data of the generator and transformer are as follows:

- GenA capacity = 100MW
- Gen–TFX rating = 150MVA
- Gen–TFX resistance = 0.001 per unit (100MVA base)

Note that both the station load and reactive power are ignored in this case for the sake of simplicity.

The flow and loss tranches for Gen–TFX can be constructed as below:

Point	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
Flow	-150	-112.5	-75	-37.5	0	37.5	75	112.5	150
Loss	0.225	0.127	0.056	0.014	0	0.014	0.056	0.127	0.225
Segment Slope	-0.002625	-0.001875	-0.001125	-0.000375	0.000375	0.001125	0.001875	0.002625	

Suppose GenA offers its full capacity at $50/MWh, then the nodal price at BB1 is equal to the GenA offer, i.e., $50/MWh.

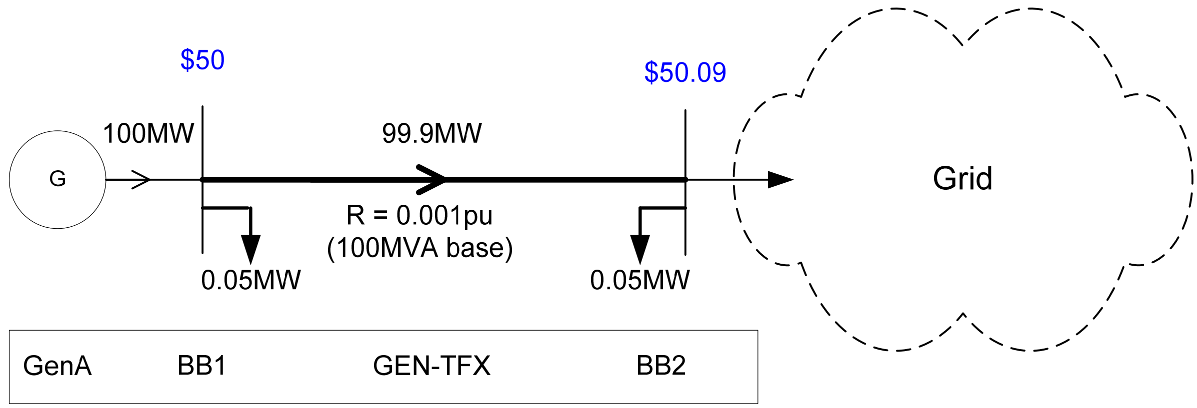
The nodal price at BB2 can be calculated based on the formulation we derived in the last section, i.e.,

Price at BB2 = Price at BB1 * (2 + SegmentSlope(i)) / (2 - SegmentSlope(i))

where *i* is the appropriate tranche.

The flow is on the tranche [7–8]. Hence the Segment Slope for this tranche is applied in the above formula to derive the price at BB2 as:

$50 * (2 + 0.001875) / (2 - 0.001875) = $ 50.09/MWh

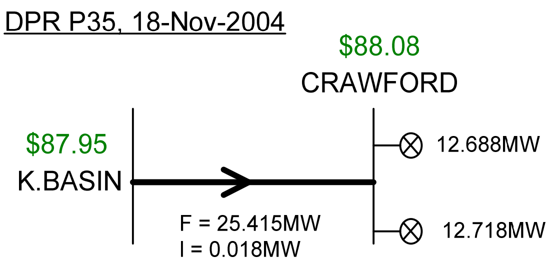


This simple example demonstrates one way for the gencos to conduct their own calculations to see how much their offer price will be increased when it enters the grid. Of course, they would have to feed in their own operating parameters.

5. An Actual Case from the NEMS

The real-time dispatch run (DPR) at period 35 on 19 November 2004 was examined to test the theory and the formulation that we derived. In light of the confidentiality of the genco data, a transmission line in the grid was used. The details are as follows:

Circuit LINE66 : CRAWFD : K.BASN 1 is the focus of this case study.



- The reported flow on this circuit is 25.415MW, with a loss of 0.018MW.
- There are two off-take loads at the CRAWFORD busbar, with consumption of 12.688MW and 12.718MW respectively.
- The reported nodal price at the K.BASIN busbar is $87.95/MWh, while the nodal price at CRAWFORD is $88.08/MWh.

As registered in the standing data, the capacity of the circuit is 80MVA, and the resistance is 0.00245 per unit. Hence, the flow and loss tranches for this circuit can be constructed as below:

Point	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
Flow	-80	-60	-40	-20	0	20	40	60	80
Loss	0.157	0.088	0.039	0.010	0	0.010	0.039	0.088	0.157
Segment Slope	-0.00343	-0.00245	-0.00147	-0.00049	0.00049	0.00147	0.00245	0.00343	

Since the nodal price at the K.BASIN busbar is $87.95/MWh, the nodal price at CRAWFORD can be calculated as below:

Price at CRAWFORD = Price at K.BASIN * (2 + SegSlope(i)) / (2 - SegSlope(i))

where i *is the appropriate tranche*

The flow is on the tranche [6-7]. Hence the Segment Slope for this tranche, i.e., 0.00147, is applied to the above formula to derive the price at CRAWFORD as:

$87.95 * (2 + 0.00147) / (2 - 0.00147) = $ 88.08/MWh

This price matches the MCE report exactly, proving that the formulation derived here can be used to replicate price discovery in the MCE.

6. Conclusion

The nodal price difference across a circuit is determined by the loss curve. However, it is not directly in proportion to the slope of the loss segment. Instead it is calculated using the following formula:

$$P_{receiving} = P_{sending} * (2 + Slope_{segment(i)}) / (2 - Slope_{segment(i)})$$

This results from the market design and the linearization of the loss model in the NEMS and does not have many physical implications. Even when they have a similar design, different markets may take different approaches resulting in different methodologies for the nodal price relationship. Hence, this relationship is specific to NEMS and should not be applied to other markets as a general rule.

However, what all electricity markets have in common, if nodal pricing is employed, is that the nodal prices increase along the power flow. This leads to locational rewards for the generating unit that is nearer to the load centre.

Glossary

genco, generating company
A company that generates electricity.

LMP, locational marginal price
A price that is calculated to reflect the location of the dispatch. Also called place-of-use pricing.

MCE, market clearing engine
The software used in the NEMS to discover dispatch schedules and prices.

MOD, merit order dispatch
Dispatch in the order of offered price. Cheaper offer gets cleared first.

NEMS, National Electricity Market of Singapore
The Singapore electricity market.